# Scientific Calculator Lessons for Middle School Teachers

A Compilation of Lessons for Implementing the 2001 Virginia Mathematics Standards of Learning



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Division of Instruction Virginia Department of Education

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# Introduction

In spring 2006, students will use scientific calculators for taking the Virginia Standards of Learning mathematics tests for grades 6, 7, and 8. The best way to prepare students to use a calculator during assessment is to give them ample practice using the tool during instruction.

The Mathematics Standards of Learning for grades 6, 7, and 8 offer many opportunities for using scientific calculators during instruction. The lessons that follow contain a range of instructional practices, from direct instruction on how to use such a calculator to writing-to-learn lessons to handson engagement. The lessons show keystrokes for the four VDOE-approved scientific calculators: the TI30Xa-SEVA, the Casio fx-260 School, and the Sharp EL-501V and EL-501WBBK.

Additionally, six lessons that extend the content of the grade 8 mathematics standards are provided for teachers to use in appropriate circumstances. Included also are four grade 8 science lessons that make use of the scientific calculator. These lessons meet or extend the grade 8 Standards of Learning for science.



# **Acknowledgments**

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# Lessons to Implement the Mathematics Standards of Learning





# 1. Charitable Cans to Paint

**Objective** Students will find the surface area of a cylinder, using the formula  $2\pi r^2 + 2\pi rh$ .

**Summary** Students gain practical experience in collecting data and calculating the surface area of cylinders.

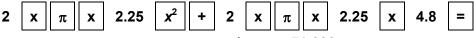
Related SOL 8.7, 7.8

Materials Scientific calculators; variety of cylindrical containers (cans); measuring tapes

### **Procedure**

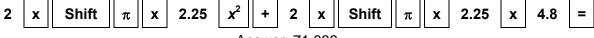
- 1. Set up the context for your lesson by announcing: "We are having a collection for needy preschool students in our area." (The collection could be for anything e.g., mittens for the homeless, dog food for the SPCA.) "We have 10 large cans to place around the school to collect school supplies for these children." You will need rather large cans, such as trash cans or popcorn tins, for collection of items such as mittens, eyeglasses, etc. Number each can for identification purposes.
- 2. Set up the math connection by explaining: "Our job is to paint each of these cans, including the lids, with our school colors. We need to figure out how much paint to buy." Use the data that one gallon of paint covers 350 square feet, and one gallon costs \$15.00.
- 3. Measure a sample can with the class, using the formula for surface area  $(2\pi r^2 + 2\pi rh)$  and the data r = 2.25 in. and h = 4.8 in. Sample keystrokes:

TI30Xa-SEVA



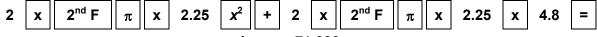
Answer: 71.039

Casio fx-260 School



Answer: 71.039

Sharp EL-501V and EL-501WBBK



Answer: 71.039

- 4. Allow students to work in pairs to calculate the surface area of each numbered can and then to determine the amount of paint they will need to buy and its cost.
- 5. When each pair has completed the task, compare all the students' results. Ask: "What would cause some answers to be different?"



# 2. Checking Up!

**Objective** Students will comprehend and apply order of operations on a scientific calculator.

**Summary** Students practice checking their calculated answers to previously solved order-of-operations problems, using a scientific calculator. Much discussion about the function of the keys occurs during the practice that reinforces the order of operations.

**Related SOL** 8.1, 7.2

**Materials** Scientific calculators; previously completed order-of-operations problems with step-by-step solutions

### **Procedure**

1. Use an example of the students' completed problems to demonstrate solving by using keystrokes on the calculator to check order-of-operations work. Demonstrate the keystrokes by displaying them on an overhead acetate or with an overhead calculator. You may want to have the students write the first few problems in keystroke notation to become familiar with it.

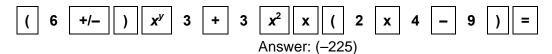
Sample problem:  $(-6)^3 + 3^2(2 \times 4 - 9)$ 

Keystrokes:

TI30Xa-SEVA



Casio fx-260 School



Sharp EL-501V and EL-501WBBK



/ ....e... ( <u>-</u>--

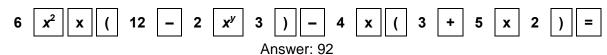
Sample problem:  $6^2(12-2^3) - 4(3+5 \times 2)$ 

Keystrokes: TI30Xa-SEVA

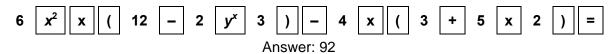
6  $x^2$  X ( 12 - 2  $2^{nd}$   $x^3$  ) - 4 X ( 3 + 5 X 2 ) =

Answer: 92

## Casio fx-260 School



# Sharp EL-501V and EL-501WBBK



- 2. Work together through various problems until the students have mastered the keystrokes.
- 3. Once students are conversant with the keystrokes, have them check various problems on their own.

3

- 4. NOTE: Common errors that students make when using the calculator are
  - forgetting the multiplication symbol before ( )
  - forgetting to press the = key at the end of a computation.



# 3. Collect Them All

**Objective** Students will collect data, represent it in a chart, calculate mean, calculate tax, and

describe the activity as a simulation.

**Summary** Students use the random number generator on the calculator to simulate the purchase

of boxes of cereal in order to "collect the whole set" of toys or prizes given by the cereal company. They make a prediction, gather data, record it on a chart, and draw

conclusions.

**Related SOL** 7.4b, 7.14, 7.16, 7.17, 7.18

**Materials** Calculators with random number generator; copies of attached worksheet; empty

cereal boxes (optional); small prizes (optional)

### **Procedure**

1. Begin the lesson by describing the context as defined in the first paragraph of the worksheet. If your students need visuals or manipulatives to understand the task clearly, use empty cereal boxes and small toys or prizes.

- 2. Discuss the question: "How many boxes do you think a person will need to buy in this situation to collect all 10 prizes?" After discussion, have students record their predictions on their worksheets.
- 3. Instruct students in the procedure for generating random numbers. Most calculators are programmed to generate a 3-digit decimal. For this simulation, the numbers will represent a specific prize in the set (i.e., .1xx will represent prize number 1, .2xx will represent prize number 2, etc.)

Keystrokes:

TI30Xa-SEVA

(No random number generator)

Casio fx-260 School

2<sup>nd</sup> Ran#

Sharp EL-501V and EL-501WBBK

2<sup>nd</sup> Random

Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72						
TI30Xa-SEVA						
75 $\Sigma$ + 16 $\Sigma$ + 34 $\Sigma$ + 29 $\Sigma$ + 72 $\Sigma$ + $\mathbf{z}^{\text{nd}}$ $\overline{\mathbf{x}}$						
To clear: ∑+ 2 <sup>nd</sup> On/AC						
Casio fx-260 School						
Enter stats mode: Mode .						
Shift SAC 75 Data 16 Data 34 Data 29 Data 72 Data Shift $\overline{x}$						
Or Shift n (for number of data points entered)						
Or Shift $\sum x$ (for the sum of the data)						
Exit stats mode: Mode 0						
Sharp EL-501V and EL-501WBBK						
Enter stats mode: 2 <sup>nd</sup> F STAT						
75 Data 16 Data 34 Data 29 Data 72 Data $\bar{x}$						
Exit stats mode: 2 <sup>nd</sup> On/C or Off						

- 4. Have students simulate a buyer and generate enough random numbers until they get at least one tally in each column (representing the different toys.) Then, have students total the number of boxes that buyer had to purchase. Have them repeat the process for 10 buyers and determine the mean number of boxes required to get all the toys.
- 5. Hold a discussion about how the simulation would be different if the manufacturer produced different numbers of toys. Do you think companies produce the same number of toys if they want buyers to collect an entire set of toys?



# Collect Them All Worksheet

ame:						Date:					
he Genera crease sa umber of e rder to coll	les. The each toy lect all 1	re are 1 in the b 0 toys?	0 differe oxes, ho Record	ent toys i ow many your pre	n this se boxes, diction l	et. Assu on aver here	ming the age, wo	compa uld a pe	ny has c erson ne	distribute ed to pu	ed an equ Irchase ir
se the ran mulation, and tenth to imulate 10 ow many e	any num by is repo differen	nber that resented nt buyers	t begins d by .0. s by tally	with .1 w	will repre	esent the	e first to	y, .2 will that ead	represe	nt the s	econd, et
Person	Toy 1	Toy 2	Toy 3	Toy 4	Toy 5	Toy 6	Toy 7	Toy 8	Toy 9	Toy 10	Total Boxes
Α											
B C											
D											_
E											
 F											
G											
Н											
<u> </u>											
J											
				of boxes stimate?	•			•	•	all 10 to	oys.
What	percent	represe	nts the t	ax on fo	od in Vi	rginia?_					Re-ST
If the	cereal co	osts \$2.5	59 per b	ox, how ? (Don't f	much m	noney w					
Suppo	se the c	ereal w	as on sa	•	veek for	only \$2	.29 per	box, plu		ow mucł	n money

## **Extension**

6. Suppose that the company did not produce an equal number of each toy. How would the situation be different?



# 4. Gifts for the Family

**Objective** Students will compute discounts.

**Summary** Students select from catalogs gifts for family members and calculate the cost to

determine whether \$200 is enough to complete the purchase.

Related SOL 8.3

**Materials** Various store catalogs; scientific calculators; copies of attached worksheet

### **Procedure**

1. Demonstrate a sample activity, such as the following:

I have \$200 to spend on my family. I found a CD player for my dad for \$89.00. I have a 15%-off coupon. How much will I spend on this gift?

Keystrokes:

TI30Xa-SEVA

89 - ( 89 x 15 2<sup>nd</sup> % ) =

Total amount spent: \$75.65

Casio fx-260 School

89 x 15 Shift % - NOTE: The Casio fx-260 School keystrokes for this work are not conducive to mathematical understanding.

Sharp EL-501V and EL-501WBBK

89 - ( 89 x 15 2<sup>nd</sup> F % ) =

- 2. Tell students that they have \$200 to spend on gifts for their family members. (You may want to use checking accounts.), and have them look through the catalogs, select items for purchase, and record their purchases (item and price) on the worksheet.
- 3. When students finish shopping, have them draw a secret discount card from a hat, basket, or box. The discounts should range from 5% off to 25% off. Let the students draw one card for each item they recorded as a purchase.
- 4. Have the students calculate the amounts spent after the discount and record these amounts on the chart. You may wish to have the students work in pairs to check each other's work.
- 5. Allow students to find an item for themselves with the total amount of money saved by using the discount cards.

### Extension

1. Consider creating tax cards as well, and add a column to the chart for the total cost of the items after the tax is added.



# Gifts for the Family Worksheet



Date: \_\_\_\_\_

Name of Item	Original Price	Discount Percent	Amount Spent
w much would you have spent on a	Il the items if there h	nad been no discour	nts?
w much did you end up spending or	n all the items by us	ing the discounts? _	
ow much money did you save by usir	ng the discounts? _		

Name:



# 5. Going Out to Lunch

**Objective** Students will compute tip, sales tax, and total bill for a meal.

**Summary** Students calculate final restaurant bills, working with data found in menus from local

restaurants.

Related SOL 7.4

**Materials** Scientific calculators; area restaurant menus; copies of attached worksheet

## Procedure

1. Give each student or group of students a menu and worksheet.

- 2. Work through the sample on the worksheet with the class.
- 3. Have students choose meals from their menu, working either individually or in groups. Then, have them calculate what their total cost is for the meal, including tax and tip.





# Going Out to Lunch Worksheet

Name:	Date:
	.37 slice of apple pie. He had good service, so he area is 5.5%. How much will he pay for his lunch?
Cost of meal: \$4.89 + \$1.37 = \$6.26 Sales tax is 5.5% of \$6.26 = \$0.34 Tip is 20% of \$6.26 = \$1.25	Add cost, tax, and tip for Bob's total amount: \$6.26 + \$0.34 + \$1.25 = <b>\$7.85</b>
Keystrokes:	
TI30Xa-SEVA	Casio fx-260 School
6.26 x 5.5 2 <sup>nd</sup> % =	6.26 x 5.5 Shift %
Answer: round 0.3443 to \$0.34	Answer: round 0.3443 to \$0.34
6.26 x 20 2 <sup>nd</sup> % =	6.26 x 20 Shift %
Answer: round 1.252 to \$1.25	Answer: round 1.252 to \$1.25
	*Remember: Tax and tip are calculated on the original cost.  nenu in front of you. You should include a meal, appetizer as well. Use the guide below to figure out
Food items and their cost:	
	Total cost:
·	nount of tax: nount of tip:



# 6. Inspecting a Deck

**Objective** Students will solve real-life problems involving right triangles by using the Pythagorean

Theorem. They also will verify the Pythagorean Theorem.

**Summary** Students play the role of building inspector, checking whether decks built in a

neighborhood are "square," and reporting their findings to the neighborhood

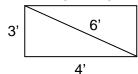
association.

Related SOL 8.10

Materials Scientific calculators; copies of attached worksheet

### **Procedure**

- 1. Present students with the problem of a building inspector checking to see if decks have been built "square" in response to the attached letter from a neighborhood association. Explain the term square in this case as defining a "quadrilateral with all angles 90 degrees."
- 2. Ask the students whether the dimensions shown for the deck below indicate that it is "square." Work through it together, using the Pythagorean Theorem.



The diagonal should be 5', but this deck has a diagonal of 6'. Therefore, this deck is not "square."

Keystrokes:

TI30Xa-SEVA

 $3 \quad \boxed{x^2 + 4} \quad \boxed{x^2 = \sqrt{x}}$ 

Casio fx-260 School

 $3 \quad x^2 \quad + \quad 4 \quad x^2 \quad = \quad Shift \quad \sqrt{x}$ 

Sharp EL-501V and EL-501WBBK

3  $x^2$  + 4  $x^2$  =  $\sqrt{\phantom{a}}$ 

- 3. Point out the use of the Pythagorean Theorem.
- 4. Give each student or group of students a copy of the letter from the neighborhood association along with the accompanying diagrams of the decks.
- 5. Have the students use the Pythagorean Theorem to compute the length of the diagonal (hypotenuse of the two triangles formed) of each rectangular deck shown and determine if each deck is "square."
- 6. Have the students put the answers to the decks into a reply letter to the neighborhood association.



**Answer key**The answers to the Inspecting a Deck Worksheet are:

- 1. No
- 2. Yes
- 3. Yes
- 4. No
- 5. Yes
- 6. Yes





# Short Pump Neighborhood Association 1111 Short Pump Road Richmond, VA 23219

September 15, 2005

Dear Building Inspector:

The residents in our neighborhood have recently built decks in their backyards. We were told that the decks should be built "square." It is our understanding that building "square" means that all corners must be right angles. We are not sure whether we did this correctly. Attached are diagrams of our decks with measurements. Please determine which decks were built "square" and inform us in writing which decks are correct and which are not.

Sincerely,

John Q. Smith, President Short Pump Neighborhood Association



# Inspecting a Deck Worksheet

Name:

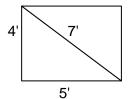
Date:



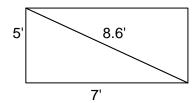
Use the Pythagorean Theorem to compute the length of the diagonal (hypotenuse of the two triangles formed) of each rectangular deck shown below, and determine if each deck is "square."

## **Sketches of Homeowners' Decks**

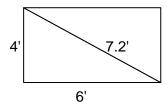
1.



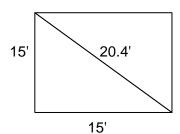
2.



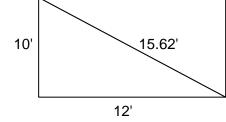
3.



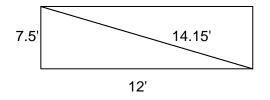
4.



5.



6.





# 7. Introducing Exponents with a Calculator

**Objective** Students will simplify numbers with positive exponents.

**Summary** Students receive direct instruction demonstrating the keystrokes used for exponential form.

Related SOL 6.22

Materials Scientific calculators

## Procedure

- 1. Review exponential notation with your students.
- 2. Demonstrate the  $x^y$  key on the calculator.

Keystrokes for calculating 74:

TI30Xa-SEVA

Casio fx-260 School

Sharp EL-501V and EL-501WBBK

- 3. Have students write each of the following as a product, using their calculators:
  - 1. 4<sup>3</sup>
- 2. 5<sup>2</sup>
- 3. 9<sup>1</sup>
- 4. 10<sup>8</sup>
- 5. 78<sup>0</sup>

- 6. 2<sup>6</sup>
- 7. 4<sup>0</sup>
- 8. 4<sup>4</sup>

# Answer key

- 1. 64
- 2. 25
- 3. 9
- 4. 100,000,000
- 5. 1

- 6. 64
- 7. 1
- 8. 256



# 8. Let the Chips Fall Where They May

**Objective** Students will gather data and represent it in a chart, find the areas of circles, find the

volumes of cylinders, calculate percentages, and calculate the mean.

**Summary** Students count the number of chocolate chips in representative chocolate chip cookies

of various brands to determine which brand contains more chips per cubic unit. They

may also calculate the brand with the most chips for the price sold.

**Related SOL** 6.12a, 6.19, 7.4, 7.8, 7.16, 7.18, 8.3, 8.7

Materials Chocolate chip cookies of 3 to 5 brands, enough so that each student or pair can have

5 cookies; metric rulers; string; 5 cups full of water for each student or pair for soaking the cookies; a graduated cylinder for each student or pair; scientific calculators; copies

of attached worksheet

### **Procedure**

This is an activity that works best in a science lab so that students have a water source readily available to them; however, this is not a requirement.

- 1. Give each student or pair of students 5 chocolate chip cookies of a single brand, and have them estimate how many chips they think each cookie contains.
- 2. Have the students take the appropriate measurements needed to calculate the area of the top of the cookie and the volume of the cookie (see chart on worksheet).
- 3. Next, have the students "melt off" the cookies from the chips by soaking them in water. Then, have them place the chips in a graduated cylinder to determine their total volume. Have the students answer the various questions on the student worksheet related to volume, percentage volume, etc. (NOTE: You may need to demonstrate how to measure the volume of an object through displacement, making sure to instruct the students in reading the level of the meniscus carefully.) For conversion purposes, give students the conversion fact 1 cubic cm = 1 ml.
- 4. If the students or student pairs have been given cookies of differing brands, have the whole class make comparisons between brands related to price, chip content, etc.



Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72
TI30Xa-SEVA
75 $\Sigma$ + 16 $\Sigma$ + 34 $\Sigma$ + 29 $\Sigma$ + 72 $\Sigma$ + $\mathbf{z}^{nd}$ $\overline{\mathbf{x}}$
To clear: ∑+ 2 <sup>nd</sup> On/AC
Casio fx-260 School
Enter stats mode: Mode .
ShiftSAC75Data16Data34Data29Data72DataShift $\overline{x}$
Or Shift n (for number of data points entered)
Or Shift ∑x (for the sum of the data)
Exit stats mode: Mode 0
Sharp EL-501V and EL-501WBBK
Enter stats mode: 2 <sup>nd</sup> F STAT
75 Data 16 Data 34 Data 29 Data 72 Data $\bar{x}$
Exit stats mode: 2 <sup>nd</sup> On/C or Off



# Let the Chips Fall Where They May Worksheet

Name:	Date:	

### **Materials**

- 5 cookies of the same brand
- Piece of string
- Metric ruler
- 5 cups, each filled about 2/3 full of water
- Spoon
- Graduated cylinder



- 1. Estimate how many chips are in each cookie. Record your estimations in column 2 of the table below.
- 2. Measure the circumference of each cookie by wrapping the string around the edge. Mark off the length of string, and measure it with the ruler, accurate to the nearest millimeter. Put your data in column 3.
- 3. Using the formula for the circumference of a circle, calculate the diameter of each cookie, and record the results to the nearest tenth of a unit in column 4. Are all the cookies the same? \_\_\_\_\_
- 4. Using the diameter that you calculated and the formula for the area of a circle, compute the area of the top surface of each cookie. Record your calculated value to the nearest tenth of a unit in column 5.

Cookie	Total Chips (estimated)	Circumference (measured)	Diameter (calculated)	Area of Top (calculated)	No. Chips on Surface (counted)	No. Chips per Square Unit (calculated)
1						
2						
3						
4						
5						

- 5. Count the number of chocolate chips visible on the top surface of each cookie, and record the total number in column 6.
- 6. Calculate the number of chips per square unit of cookie for each of your cookies. Record the data in column 7.
- 7. Calculate the mean number of chips per square unit of cookie for your brand of cookies.



Cookie	Area of Top (calculated)	"Height" (measured)	Volume (calculated)	Total Volume of Chips	No. Chips per Cubic Unit	Volume of Chips per Cubic Unit	% Total Volume from Chips
1							
2							
3							
4							
5							

8.	Copy your data for the area of the top of the cookie into column 2 of the table above.
9.	Measure to the nearest tenth of a unit the "height" of the cookie. Record your data in column 3.

10.	Explain why a chocolate chip cookie is considered a cylinder.								

- 11. Using the formula for the volume of a cylinder, calculate the volume, to the nearest tenth of a unit, of each cookie. Record your data in column 4 in the table above.
- 12. Since the chocolate chips are not a standard geometric shape, you'll need to find their volume by displacement. To remove the cookie from around the chips, soak each cookie in its own cup of water until the cookie is soft enough to fall away from the chips. Use a spoon to stir the water to help dissolve the dough and to get the chips out of the water when they are free.
- 13. Being careful to keep the chips from each cookie separate from the others, count the number of chips in each cookie. How did the actual number compare to your earlier estimate?
- 14. Fill the graduated cylinder about half full. Carefully drop the chips from cookie 1 into the cylinder to measure their total volume. Record this data in column 5. Once you have finished, pour out the water and the chips. (Don't let the chips go down the sink: they should go into the trash can.) Repeat the procedure for each of the other four cookies.
- 15. Using the data from #11, determine the number of chips per cubic unit of cookie. Record the data in column 6.
- 16. Using the data from #12, determine the volume of chips per cubic unit of cookie. Record the data in column 7.
- 17. Calculate the percentage of each cookie that consists of chocolate chips, and record the data in the last column.
- 18. Calculate the mean of the percentages of chip volume for your brand of cookie. \_\_\_\_\_



## **Class Data**

19. Share all data with your classmates for the different brands of cookies represented. Record the data in the table below.

Brand of Cookie	Price per Bag	No. of Cookies per Bag	Price per Cookie	Mean No. Chips per Cookie	Mean % Chip Volume per Cookie	% Chips per Unit Price

- 20. Which brand of cookie provides the most chips per cookie, on average?
- 21. Which brand of cookie provides the most chips by volume per cookie, on average?
- 22. Which brand of cookie provides the highest percentage of chips by volume per cookie, on average?
- 23. Assuming you were a chocolate chip lover, which brand of cookie would you be most likely to buy? \_\_\_\_\_ Why?



# 9. How to Avoid Making a Mess with pi

Objective Students will calculate the circumference of a circle, recognize the difference in

answers when using the pi key versus 3.14, and use the "fix" function to set pi at 3.14.

**Summary** Students compare and contrast the effect on answers to problems of pi rounded to

various numbers of decimal places. This is specifically related to the internal rounding

features of scientific calculators.

Related SOL 6.12

**Materials** Overhead projector; scientific calculators

### **Procedure**

NOTE: This lesson should involve student discussion at each step.

- 1. Draw a circle, and define the word *circumference* as the distance around a circle.
- 2. Present the formulas ( $C = \pi d$  and  $C = 2\pi r$ ) for finding circumference to the students, and discuss substituting the values. For example, if the radius of the circle is 3 m, and the value of  $\pi$  is 3.14, an approximation of the circumference of the circle is:  $2\pi r = (2)(3.14)(3 \text{ m}) = 18.84 \text{ m}$ .
- 3. Repeat the exercise, using the pi key on the calculator instead of entering 3.14 as the value. Keystrokes:

TI30Xa-SEVA

2  $\pi$   $\pi$   $\pi$  3 = 18.84955592

Casio fx-260 School

2 x Shift  $\pi$  x 3 = 18.84955592

Sharp EL-501V and EL-501WBBK

2  $\mathbf{x}$   $\mathbf{2}^{\text{nd}} \mathbf{F}$   $\pi$   $\mathbf{x}$  3 = 18.84955592

- 4. Ask the class why the answer using 3.14 differs from that using pi in the calculator. Discuss the fact that pi on the calculator is not rounded to only 2 decimal places, but to 9 decimal places. Ask which answer is more precise. Explain that, in the real world, people most often use pi rounded to two decimal places to calculate circumference. Why would someone use pi to 9 decimal places?
- 5. Have students compare the two answers, noticing that 18.84 is not 18.84955592 rounded to the nearest tenth. Why?



6. Show students how to "fix" the decimal places of their calculator to get an answer that is already rounded, using the more standard form of pi.

Keystrokes:

TI30Xa-SEVA

To fix display to two decimal places:

2<sup>nd</sup> fix 2

Casio fx-260 School

Mode 7 2

Your calculator screen should read 0.00

To clear fixed decimal display

Mode On

Sharp EL-501V and EL-501WBBK

To fix display to two decimal places:

2<sup>nd</sup> F TAB 2

Your calculator screen should read 0.00.

To return to floating decimal display

2<sup>nd</sup> F TAB .

- 7. Have students key in their problem, using the same keystrokes as before. Make sure they notice that pi comes up as 3.14, but that the answer on their screen is 18.85. Ask why this answer is different from the 18.84 answer they received when they keyed in 3.14 as pi.
- 8. Allow students to try different problems to see how a rounded pi affects answers. Is there a difference if 2 and 3 are not among their values?



# 10. Shopping for Groceries

**Objective** Students will compute a discount and resulting sale price of one discount.

**Summary** Students calculate the cost of the meal that they plan, using grocery store ads and

discount coupons.

**Related SOL** 7.4, 8.3

Materials Grocery store ads from newspapers; scientific calculators; coupon slips; copies of the

two attached worksheets

### Procedure

1. Place students into groups of four.

- 2. Give each student a different discount coupon with each group receiving one for 10% off, one for 15% off, one for 20% off, and one for 25% off.
- 3. Explain to the students that they are going to plan a meal. Give them grocery store circulars/ad with prices and Worksheet A.
- 4. Do the sample problem together, using a calculator to illustrate appropriate keystrokes.
- 5. Have students choose their food items together as a group. They must choose a meat, two side dishes, and a dessert. Have them record their choices on worksheet.
- 6. Have students find the discounted price of each item, using their individual discount card.
- 7. As the groups finish, give them Worksheet B. Explain that this task is for the group to plan the most economical meal, using their coupons to maximum advantage.
- 8. When all groups are done, find out which group saved the most money on their meal.



# Shopping for Groceries Worksheet A

Name:	Date:
SAMPLE PROBLEM: Squeaky Clean shampoo regularly coone item. What will the cost of the shampoo be? (Do not inc	
Find the percentage off, using the following keystrokes:  TI30Xa-SEVA  \$2.69  x  18  2 <sup>nd</sup> %  =  Casio fx-260 School  \$2.69  x  18  Shift  %	
Sharp EL-501V and EL-501WBBK  \$2.69	
Then, subtract: \$2.69 – \$0.48 = <b>\$2.21</b> (Alternatively, you may find 82% of \$2.69 and eliminate the	• ,
My discount card is for % off.	
Meat: Cost: Amount off: Price after discount:	
Side dish 1: Cost: Amount off: Price after discount:	
Side dish 2: Cost: Amount off: Price after discount:	
Dessert: Cost: Amount off: Price after discount:	



# Shopping for Groceries Worksheet B

Group Name:	Date:
Meat:	
Original cost:	
Percent of discount:	
Amount of discount:	
Price after discount:	
Side dish 1:	
Original cost:	
Percent of discount:	BBB BBB BBB BBB BBB BBB BBB BBB BBB BB
Price after discount:	The second second
Side dish 2:	
Original cost:	
Percent of discount:	
Amount of discount:	
Price after discount:	
Dessert:	
Original cost:	
Percent of discount:	
Amount of discount:	
Price after discount:	
Original cost of meal:	
Your cost after discounts:	-
Total amount of money saved:	
*Go back and change who buys which item, a	nd compare the total amount of money you saved.
Describe a strategy for using your coupons to	save the most money.



# 11. Who Are You Calling a Nerd?

**Objective** Students will gather data and represent it in a chart and in a stem-and-leaf plot, and to

find the mean, median, and mode of the data.

**Summary** Students count Nerds™ candy and, with the aid of a scientific calculator, determine the

values of the measures of central tendency (mean, median and mode) for the data.

They also make conjectures based on their data and discuss the concepts of

operational definition and outliers in a data set.

**Related SOL** 6.18, 6.19, 6.20.a, 7.16, 7.17

Materials One Nerds™ snack-size candy box for each student (these can easily be found in mid-

to late-October in bags containing both grape- and strawberry-flavored candies);

scientific calculators; copies of attached worksheet

### **Procedure**

- 1. Provide each student with a snack-size box of Nerds™ candy. Allow them to open the box and spread the candy out on a piece of paper, but do not allow them to eat any yet.
- 2. Explain that in scientific research, experimenters typically have to formulate an "operational definition" of a concept for the purposes of their study. For example, *intelligence* may be defined as what an intelligence test measures. Introduce and describe the term *operational definition*, and have the class agree on what would constitute a good operational definition of *Nerd*™ for this study. Ask such questions as: Will you count a piece of candy as a Nerd™ only if it measures a specific diameter? Will a piece count as two Nerds™ if it clearly looks like two fused pieces? Does a piece have to be a certain color to be a Nerd™?
- 3. Once the operational definition of a Nerd™ is established, have each student count the number of Nerds™ in his/her box and report that number to the class to record on the data chart. Have each student record the data from the entire class in the charts provided on the worksheet and then create a stem-and-leaf plot for the strawberry Nerds™ and one for the grape Nerds™. (Consider back-to-back stem-and-leaf plots for this data.)
- 4. Have students enter the data sets in the calculator to determine the mean of each flavor, and have them determine the median and the mode from the stem-and-leaf plots they created.
- 5. Have a class discussion of outliers and their effect on the mean as opposed to the median and the mode. (The mean is sensitive to the actual magnitude of the data value as opposed to its location or to the number of times it occurs.)
- 6. Have students formulate their own questions about Nerds™ that can be answered from the stem-and-leaf plot.



Keystrokes to calculate an arithmetic mean of 75, 16, 34, 29, 72
TI30Xa-SEVA
75 $\Sigma$ + 16 $\Sigma$ + 34 $\Sigma$ + 29 $\Sigma$ + 72 $\Sigma$ + $\mathbf{z}^{\text{nd}}$ $\mathbf{x}$
To clear: ∑+ 2 <sup>nd</sup> On/AC
Casio fx-260 School
Enter stats mode: Mode .
Shift SAC 75 Data 16 Data 34 Data 29 Data 72 Data Shift $\overline{x}$
Or Shift n (for number of data points entered)
Or Shift ∑x (for the sum of the data)
Exit stats mode: Mode 0
Sharp EL-501V and EL-501WBBK
Enter stats mode: 2 <sup>nd</sup> F STAT
75 Data 16 Data 34 Data 29 Data 72 Data $\bar{x}$

Exit stats mode:

On/C

Off

or



# Who Are You Calling a Nerd? Worksheet

1.	What is an "operational definition?"		
2.	What is the operational definition of <i>Nerd</i> ™ for this class activity?		
3.			
	Strawberry Nerds™ Grape Nerds™		
4.	Create a stem-and-leaf plot for each of the data samples you gathered.		
5.	Enter the data into the scientific calculator and report the mean number of Nerds™ of each flavor. Round each value to the nearest tenth.		
	Mean <sub>Strawberry</sub> = Mean <sub>Grape</sub> =		
6.	Using the stem-and-leaf plots you created, give the values of the median and the mode for each flavor of candy.		
	Median <sub>Strawberry</sub> = Median <sub>Grape</sub> =		
	Mode <sub>Strawberry</sub> =		
7.	An "outlier" is a data value that is very different from the others values in the data set. Does either flavor appear to have any outliers? If so, which values are they?		
	Suspected outliers Strawberry: Suspected outliers Grape:		
8.	Which of the measures of central tendency were most affected by outliers that you identified in #7? Why do you think so?		
9.	What questions about Nerds™ could you answer from the stem-and-leaf plot?		

Date: \_\_\_\_\_



# 12. Who Won the Race?

**Objective** Students will devise and compare a variety of methods for determining mean, median,

and mode.

**Summary** Students devise as many ways as possible to determine which team of runners wins a

race, which places second, which third, and which fourth. All methods of determining the overall team position must be supported with logical reasoning and appropriate mathematical calculations, when appropriate. As the final project, groups present

reports on their conclusions.

**Related SOL** 8.3, 8.12

Materials Copies of attached worksheet; scientific calculators

### **Procedure**

Distribute the worksheet outlining the problem description and providing the data.

- 2. Have students read the introduction and directions. Answer any questions, especially those that relate to interpreting the data table.
- 3. Have students work individually for 10 minutes to devise as many ways as they can to determine overall team placement. After 10 minutes, have students pair up, compare their proposed methods with their partner, and together attempt to devise more ways to determine the overall winner. Have them work with these partners for about 15 minutes.
- 4. After the pairs have worked together for 15 minutes, let pairs join one another to make groups of four. Have each group's members share all of their methods with each other and then pick the one they believe is the most fair to present to the Judging Committee.
- 5. Instruct each group to prepare a brief presentation for the Judging Committee. The presentation should include the reason(s) for the choice of methods (with logical and mathematical support) and should include a comparison to other methods showing why the chosen method is preferable.

## Possible student responses

- Look at the first row (the order in which the first person from each team crossed the line): RMS
  had the fastest runner, WMS the second fastest, JMS the third fastest, and LMS the fourth
  fastest. Hence, they place: RMS, WMS, JMS, and LMS.
- 2. Find the average of the top 10 runners from each school. In this case, they finish JMS (19.2), LMS (19.8), RMS (20.9), and WMS (22.2).
- 3. Find the average for all runners on each team. Will this favor teams with more or fewer runners?
- 4. Compare the median finisher for each school. Since row 17 is the last row with a finisher from each school, consider row 9 to be the median: places are then JMS, RMS, LMS, and WMS.

### Assessment

An open-ended problem is best assessed with the use of a rubric. You will need to determine the weight of each component of the rubric, depending on the emphasis you place on each part of the activity. Suggested items to measure include the following:



- Use of correct mathematics (e.g., mean, median, mode)Fluency (lots of methods devised)
- Creativity (methods substantially different from each other)



# Who Won the Race? Worksheet



Jefferson Middle School (JMS), Washington Middle School (WMS), Lincoln Middle School (LMS), and Roosevelt Middle School (RMS) competed for the State Championship in Cross Country. The results of the race are displayed in the table at right, which shows how each runner placed compared to his/her own teammates (Within Team Rank), as well as how each runner placed overall. For example, the fastest runner from Jefferson Middle School placed 4<sup>th</sup> overall; the last runner to finish from Roosevelt Middle School finished 64<sup>th</sup> overall. Although each team entered 20 runners, 6 runners did not complete the race; therefore, no times are given for them.

When the race results were examined, the Judging Committee disagreed over which team had actually won the race. One judge thought that RMS was the winner, because a runner from that school finished first. Other judges disagreed, citing a variety of reasons.

The Judging Committee has contracted your consulting firm to devise a way to determine

Within	JMS	LMS	RMS	WMS
Team	Overall	Overall	Overall	Overall
Rank	Finish	Finish	Finish	Finish
1	4	6	1	2
2	9	7	3	5
3	11	10	14	8
4	12	13	18	15
5	20	16	19	17
6	21	22	23	31
7	25	24	28	33
8	26	27	30	36
9	29	34	32	37
10	35	39	41	38
11	43	40	44	46
12	45	42	47	51
13	49	48	50	55
14	54	52	56	57
15	61	53	60	58
16	65	62	63	59
17	69	66	64	67
18	70	72	_	68
19	71	_	_	73
20	_	_	_	74

the overall winning team. You will need to present a method to them tomorrow that you think is the best and fairest way of determining the overall winner. You must support your method with solid reasoning and mathematics. Your presentation should not only describe the best method, but should also include reference to other methods and a description of why your chosen method is preferred.

Your consulting firm consists of four people. In the past, you have found the following procedure works best:

- Individuals work alone for 10 minutes, brainstorming ideas and making initial calculations and recommendations.
- Pairs form, compare initial ideas, and then brainstorm more ideas.
- The whole firm comes together to discuss results of the work, then makes a final decision, and plans the presentation.

# Lessons to Extend the Mathematics Standards of Learning





# 13. Brief Introduction to Trigonometry

**Objective** Students will create similar triangles to investigate basic trigonometric

relationships, evaluate the three basic trigonometric ratios for acute angles in right triangles, use the calculator to evaluate sine, cosine, and tangent for a given

angle, and determine the relationship between the "co-" functions.

**Summary** Students discover various trigonometric relationships. The activity introduces

them to the sine, cosine, and tangent ratios. The lesson also involves similar triangles and illustrates that the values of the trigonometric ratios are independent

of the size of the triangle.

**Prerequisite SOL** 6.2, 7.11, 8.6, 8.10

**Materials** Copies of attached worksheet; graph paper; rulers; protractors; and scientific

calculators

# **Procedure**

1. Have students follow the instructions on the worksheet. Answer questions as needed.

- 2. You may want to follow up this exercise by rotating the triangles in space (so that the triangles in question do not always have a standard angle position) to illustrate that the relationships are true regardless of orientation in space.
- 3. You may also want to lead a discussion that tests students' number sense by investigating the values that are possible for sine and cosine in a right triangle. (0 < value < 1) Since the hypotenuse has to be the longest side of the right triangle, the denominator has to be larger than the numerator.
- 4. Continue this discussion by investigating the tangent values. (value > 0)



# Brief Introduction to Trigonometry Worksheet

Date: С On graph paper, draw a right triangle in which one leg is 1 unit long and the other leg is 1.5 units long. Label the points as shown at right. 1. Use a protractor to measure ∠CAB. 1.5 units 2. Use a protractor to measure ∠ACB. \_\_\_\_\_ В Ε Extend the leg AB by an amount equal to its length and call the new endpoint D, as shown at right. Make  $\overline{DE}$  twice as long as  $\overline{BC}$ . Draw  $\overline{CE}$ . 3. Use a protractor to measure ∠AED. \_\_\_\_\_ 4. What is the measure of ∠EAD? С What is the measure of ∠EDA? 5. 3 units 6. What is true about the two triangles that you have drawn? 1.5 units 1 unit D G Extend the leg  $\overline{AD}$  by an amount equal to the length of  $\overline{AB}$ , and call the new endpoint F, as shown at right. E How long will you have to make  $\overline{\mathsf{FG}}$ ? 7. 8. Measure ∠AGF with a protractor. 9. What is true about the three triangles you have drawn? С 3 units 1.5 units B |1 unit



The **tangent** of an acute angle in a right triangle is defined as **the ratio** of the length of the leg opposite it to the length of the leg adjacent to it.

10.	Find the tangent of ∠CAB in ∆CAB
11.	Find the tangent of ∠EAD in ∆EAD
12.	Find the tangent of ∠GAF in ∆GAF
13.	What does this tell you about the tangent of an angle relative to the size of the triangle to which it belongs?
14.	Use the calculator to find the tangent of ∠CAB
15.	Use the Pythagorean Theorem to find the length of $\overline{AC}$ Of $\overline{AE}$
	Of AG
	sine of an angle is defined as the ratio of the length of the leg opposite it to the length of the otenuse.
16.	Find the sine of $\angle CAB$ in $\triangle CAB$ .
17.	Find the sine of $\angle$ EAD in $\triangle$ EAD
18.	Find the sine of $\angle$ GAF in $\triangle$ GAF
19.	What does this tell you about the sine of an angle relative to the size of the triangle to which it belongs?
20.	Use the calculator to find the sine of ∠CAB
21.	Find the sine of $\angle$ ACB in $\triangle$ CAB
22.	Find the sine of $\angle$ AED in $\triangle$ EAD
23.	Find the sine of $\angle$ AGF in $\triangle$ GAF
24.	What does this tell you about the sine of an angle relative to the size of the triangle to which it belongs?
25.	Use the calculator to find the sine of ∠ACB
	cosine of an angle is defined as the ratio of the length of the leg adjacent to it to the length ne hypotenuse.
26.	Find the cosine of ∠CAB in ∆CAB
27.	Find the cosine of ∠EAD in ΔEAD
28.	Find the cosine of ∠GAF in ∆GAF
29.	What does this tell you about the cosine of an angle relative to the size of the triangle to which it belongs?
30.	Use the calculator to find the cosine of ∠CAB
31.	Find the cosine of ∠ACB in ∆CAB
32.	Find the cosine of ∠AED in ∆EAD
33.	Find the cosine of ∠AGF in ∆GAF
34.	What does this tell you about the cosine of an angle relative to the size of the triangle to which it belongs?



35.	Use the calculator to find the cosine of ∠ACB
36.	What is true about sin∠CAB and cos∠ACB?
	About sin∠EAD and cos∠AED?
	About sin∠GAF and cos∠AGF?
37.	What is true about m∠CAB and m∠ACB?

- 38. The prefix *co* refers to a relationship between the sine and the cosine ratios. Investigate the measurements of the angles, and make a conjecture about what *co* refers to in this context.
- 39. Test your hypothesis by selecting angles that satisfy the relationship and checking them with your calculator.



# 14. Building Towers

**Objective** Students will expand upon the basic counting principle to find the number of

arrangements, using factorials.

**Summary** Students use colored cubes to find how many ways they can be arranged. From

a chart that the class creates, students work to discover the meaning of the term

factorial.

Prerequisite SOL 7.15

Materials Five different-colored cubes for each student; scientific calculators

#### **Procedure**

1. Give each student a blue cube. Ask them how many different color orders they could have when building a tower with this one cube. (The answer is 1.)

- 2. Give each student a second color cube (the same for all). Ask how many different color orders they could have when building a tower with these two cubes. (The answer is 2.)
- 3. Give them a third color cube. Now, it's more difficult, so ask students to build towers with the three different color cubes and make all of the possible orders of colors. How many different color orders could there be? (The answer is 6.)
- 4. Give them a fourth color cube. Have the students walk around with their cubes arranged differently so that they can see all of the possible combinations. (If there are fewer than 24 students in the class, you will have to have some students holding two towers.) (The answer here is 24.)
- 5. Give them a fifth color cube, and have them guess from their seats how many different orders they could have.
- 6. Stop them in a reasonable amount of time and put up a chart on the board or overhead with the information collected:

1 cube — 1 order

2 cube — 2 orders

3 cube — 6 orders

4 cube — 24 orders

7. Discuss any patterns they may see. If they do not see any patterns, say: "Look at the combination of four colors:

How many choices did you have for the first cube? (4)

How many for the second? (3)

How many for the third? (2)

How many for the fourth? (1)

Use the Basic Counting Principle to determine the possible arrangements:

$$4 \times 3 \times 2 \times 1 = 24$$

Check for the three- and two-block towers:

$$3 \times 2 \times 1 = 6$$
  $2 \times 1 = 2$ 

8. Define factorial for students, and discuss its uses.

9. Have students practice finding factorials, using a calculator:

Keystrokes for 4!:

TI30Xa-SEVA

Casio fx-260 School

Sharp EL-501V and EL-501WBBK

10. Have students complete the chart on the board for combinations of up to 10 different colored cubes, using a calculator.



# 15. Candy? Probably

**Objective** Students will determine the difference between combinations and permutations,

determine the probability of a future event based on experimental data, list the possible arrangements of various objects in combination and permutation, and use the calculator to determine the values of combinations and permutations.

**Summary** Students investigate probabilities and determine the difference between

combinations (where order is unimportant) and permutations (where order is

significant), using M&M's™ and Skittles™.

Prerequisite SOL 6.20b, 7.15, 7.18, 8.11

**Materials** M&M's™, Skittles™, copies of attached worksheet

#### **Procedure**

1. Give each student a sample of M&M's™ and a sample of Skittles™. Have them count the candies and tally the results. Then, have them follow the procedures outlined on the worksheet to investigate the probabilities, combinations, and permutations. (The candies can be used as manipulatives in creating the combinations and permutations for those who have difficulty determining them symbolically.)

Keystrokes:

TI30Xa-SEVA

Casio fx-260 School

Permutations for 5 choose 4: 5 Shift nPr 4 =

Combinations for 5 choose 4: | 5 | | Shift | | nCr | | 4 | = |

Sharp EL-501V and EL-501WBBK

Keys are not available on these models.

2. Follow up with discussion about other examples of combinations and permutations.



# Candy? Probably Worksheet

Name: Date:								
Cou	nt the number	of candies of	each color	you have in	your sampl	e, and recor	d the data bel	OW.
	M&M's™:	Brown	Red	Orange	Yellow	Green	Blue	
	Skittles™:	Red	Orange	Yellow	Green	Purple		
Prob	<b>pabilities</b> If you random	ly select a c	andy, what o	color are you	ı most likely	/ to get?		
2.	If you random	nly select one	e M&M™, wh	nat is the pro	bability tha	t you will get	a red one? _	
3.	If you random	nly select one	e M&M™, wh	nat is the pro	bability tha	t you will get	a blue one?	
4.	If you random	nly select one	e M&M™, wh	nat is the pro	bability tha	t it will be a	green one?	
5.	If you random	nly select one	e Skittle™, w	hat is the pr	obability tha	at it will be a	n orange one	?
6.	If you random	nly select one	e Skittle™, w	hat is the pr	obability tha	at you will ge	et a purple one	e?
7.	If you random	nly select one	e candy, who	at is the prob	ability that	you will get	a yellow one?	
8.	If you random	nly select one	e candy, who	at is the prob	ability that	you will get	an orange one	e?
9.	If you random	nly select one	e candy, who	at is the prob	ability that	you will get	a green one?	
10.	If you random	nly select a c	andy and it i	is blue, what	is the prob	ability that it	is an M&M™	?
11.	If you random	nly select a c	andy and it i	is green, wh	at is the pro	bability that	it is a Skittle™	۸?
12.								
Con	binations							
13.	How many wavarious group	•	get a group	of 4 M&M's <sup>⊤</sup>	<sup>м</sup> from amo	ng the 6 colo	ors? Lis	st the
14.	How many wa	ays can you	get a group	of 3 Skittles	™ from amo	ong the 5 col	ors? List the ς	groups

In the previous two exercises, the order in which you arranged the colors did not matter: Red-Yellow-Green and Green-Yellow-Red are considered the same *combination*. Your calculator can evaluate *combinations*, using the *n*C*r* key. Evaluate 5C3 from #14 above. Go back to your list of combinations to see if you forgot any. If you did, include them below.

# **Permutations**

Suppose, when you gathered up the candies, the order in which you selected them made a difference so that Red-Yellow-Green is a different group from Yellow-Red-Green. This is called a *permutation*. (In fact, your locker combination is actually misnamed, since you have to dial the numbers in a particular order; you can't just dial the three digits in any order. Technically, it should be called a locker "permutation.")

lock	er "permutation.")
15.	How many permutations are possible using 2 colors from among the 5 Skittles™ colors? List them below.
16. 17.	Your calculator can evaluate permutations, using the <i>n</i> P <i>r</i> key. Evaluate 5P2: Go back and examine your list to see if you forgot any. If you did, find them and include them below.
18. 19.	How many permutations are possible using 4 colors from among the 5 Skittles™ colors? How many permutations are possible using 5 colors from among the 6 M&M's™ colors?



# 16. Factorial (!) or Fiction?

**Objective** Students will use the factorial key on the calculator to determine the values of 0!

through 12!. They also will examine the factorial values to determine the definition of n! and evaluate various expressions involving factorials, using the definition.

**Summary** Students generate a chart of the values of the factorials for 0 through 12 and then

41

discover the formula for n!.

**Prerequisite SOL** 6.21, 7.19, 8.14

Materials Scientific calculators capable of calculating factorials through 12; copies of

attached worksheet

# **Procedure**

1. Instruct students on how to use the factorial key on their calculators.

Keystrokes for finding 6!:

TI30Xa-SEVA

Casio fx-260 School

Sharp EL-501V and EL-501WBBK

- 2. You may need to lead the students through the beginnings of the patterns to demonstrate how to arrive at the formula n! = n(n-1)!
- 3. Have students follow the instructions on the accompanying worksheet.

Number (n)	<i>n</i> !		
0	1	1	1
1	1	1 x 1	1 x 0!
2	2	2 x 1	2 x 1!
3	6	3 x 2 x 1	3 x 2!
4	24	4 x 3 x 2	4 x 3!
5	120	5 x 4 x 6	5 x 4!
6	720	6 x 5 x 24	6 x 5!
7	5040	7 x 6 x 120	7 x 6!
8	40320	8 x 7 x 720	8 x 7!
9	362880	9 x 8 x 5040	9 x 8!
10	3628800	10 x 9 x 40320	10 x 9!
11	39916800	11 x 10 x 362880	11 x 10!
6! 228!	d479001608	91 12 x #14x 36288009	12∖x7!1!

# Answer key

Answers to #5 on the worksheet: a. 10!



# Factorial (!) or Fiction? Worksheet

Name:	Date:
	2 4.44 .

Use the factorial key on your calculator to complete the chart below.

Number (n)	<i>n</i> !		
0	1	1	
1	1	1 x 1	
2	2	2 x 1	
3	6	3 x 2 x 1	
4			
5			
6			
7			
8			
9			
10			
11			
12			

- 2. In the third column above, rewrite each n! value as the product of at most three other factors as suggested by the entries provided.
- In the last column, rewrite each n! value as a product of one integer and another n! value. 3.
- 4. Given your answers above and the fact that 0! = 1, write the formula for n!

- 5. Evaluate each of the following:
  - 10 x 9!
- b. 6 x 5! c. 8 x 7!
- d. 14 x 13!

- e. 10! ÷ 10
- f. 5! ÷ 5
- g. 20! ÷ 20
- h. 9! ÷ (9 x 8)



# 17. Half Some Candy

**Objective** Students will gather data and represent it in a chart and in a graph, use the

calculator to evaluate exponential equations, and compare experimental and

theoretical data.

**Summary** Students use the calculator to find the values of the functions  $y = 2^x$  and  $y = 0.5^x$ 

and then plot the points to create the graphs of the two functions. They then use

M&Ms™ or Skittles™ to simulate a half-life experiment that follows the

mathematical model given by y = 0.5

**Prerequisite SOL** 7.14, 8.1, 8.14, 8.18

Materials M&M's™ or Skittles™; small paper drinking cups; scientific calculators, copies of

attached worksheet

### **Procedure**

1. Begin with a review of equations and their graphs, using the worksheet.

2. Have the students complete #1 and #2 on the worksheet as examples of nonlinear graphs. Have a class discussion of how linear equations differ in form from nonlinear equations.

Keystrokes:

TI30Xa-SEVA

 $2^x$  where x = 2

 $2^x$  where x = 3

 $2^x$  where x = 4

2 y<sup>x</sup> 2 =

2 y<sup>x</sup> 3 =

2 y<sup>x</sup> 4 =

#### Casio fx-260 School

Because the key on the Casio calculator is labeled  $x^y$ , you may want to alter the letters used as variables on the worksheet.

 $2^y$  where y = 2

 $2^y$  where y = 3

 $2^y$  where y = 4

2 x<sup>y</sup> 2 =

 $2 \quad x^y \quad 3 \quad = \quad$ 

 $2 \quad x^y \quad 4 \quad = \quad$ 

# Sharp EL-501V and EL-501WBBK

- $2^x$  where x = 2
- $2^x$  where x = 3
- $2^x$  where x = 4
- 2 y<sup>x</sup> 2 =
- 2 y<sup>x</sup> 3 =
- 2 y<sup>x</sup> 4 =
- 3. Distribute the candies and cups. Have students read the procedure in step #4 on the worksheet. Answer any questions to help clarify the procedure before letting the students continue with the remainder of the activity.
- 4. If students have not been exposed to the concept of half-life, one definition is "the period of time it takes for half of a radioactive substance to decay." (Consider consulting with the eighth-grade science teacher on this lesson for a math/science connection.)
- 5. End the lesson with a class discussion of question #10 from the worksheet and the difference between experimental and theoretical values.

# **Extension**

If you wish, you could expand this concept to include the topics of domain and range, and independent and dependent variables.

# Half Some Candy Worksheet

Name:	Date:	
ivaille	Date	

1. Use the calculator to find the values of the function  $y = 2^x$ . Complete the table of values, and then draw the graph.

X	y = 2 <sup>x</sup>
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

2. Use the calculator to find the values of the function  $y = .5^x$ . Complete the table of values, and draw the graph.

x	$y = .5^{\times}$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
1	

- 3. Record the number of candies you have in your sample. \_\_\_
- 4. Shake the cup and gently toss the candies out onto a piece of paper. Record the number of candies that landed "face up" (that is, with the letter of the candy showing). \_ landed "face down" will not be used again. (You may just have to eat those! ©)
- 5. Complete the table below, and then plot the data values at right.

Toss#	Candies Left
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

8		
9		

- 6. Which of the graphs in steps #1, #2, or #5 does your model most resemble? \_\_\_\_\_
- A "perfect" half-life model follows the equation  $y = A_0(1/2)^x$ , where  $A_0$  equals the number in the 7. original sample.

Write the perfect half-life equation for your sample.

8. Use your calculator to evaluate the perfect half-life values for your candy sample. Complete the chart, and graph the perfect half-life curve.

	•
X	$y=A_0(1/2)^x$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

- 9. How do your actual values compare to the perfect half-life situation?
- 10. Write a paragraph to explain why you think your experimental values do not match the theoretical, perfect half-life equation values. What about the situation makes it nearly impossible to re-create the perfect equation values? What other sources of error might there be?



# 18. Way Off Base!

**Objective** Students will change numerals from base 10 to other bases.

**Summary** Students change numbers from base 10 to other bases, namely 2 (the binary

system) and 8 (octal). More advanced students may also convert numbers to base 16 (hexadecimal, a system that uses the digits 0 through 9 and the letters A

through F).

Prerequisite SOL 8.1, 8.5

**Materials** Scientific calculators, copies of attached worksheet

# Procedure

- 1. Using expanded notation and numbers through 100,000, review with students why our number system is a called a base 10 system.
- 2. Compare our base 10 system to a base 2 system, using the expanded notation format. Point out to students how a chart, such as the one they are about to complete, can help in converting numbers to different bases. Additional information, such as the way the binary system is the language of computers, is important for students to see relevance in the activity.
- 3. Once the students have a fundamental understanding of bases, allow them to work individually, in pairs, or in small groups to complete the activity.

Keystrokes for calculating 2<sup>5</sup>:

TI30Xa-SEVA

Casio fx-260 School

NOTE: If using the *Casio fx-260 School*, you may want to change the exponent on the worksheet to 
$$y$$
.

Sharp EL-501V and EL-501WBBK

4. The values for the initial chart are as follows:

X	2 <sup>x</sup>	8 <sup>x</sup>	10 <sup>x</sup>	16 <sup>x</sup>
0	1	1	1	1
1	2	8	10	16
2	4	64	100	256
3	8	512	1,000	4,096
4	16	4,096	10,000	65,536
5	32	32,768	100,000	1,048,576
6	64	262,144	1,000,000	16,777,216



**Answer key**The answers to the questions on the worksheet are as follows:

2.	a.	111001	b.	1000111	C.	1000	d.	100111	e.	10001
3.	a.	71	b.	107	C.	1102	d.	47	e.	144
4.	a.	6	b.	13	C.	22	d.	55	e.	91
5.	a.	6	b.	15	c.	26	d.	67	e.	133
6.	a.	36	b.	79	C.	12C	d.	1388	e.	15D77

# Way Off Base! Worksheet

Name:	Date:
-------	-------

1. Using the  $y^x$  key on your calculator, determine the values of each of the following:

				~
X	2 <sup>x</sup>	8 <sup>x</sup>	10 <sup>x</sup>	16 <sup>x</sup>
0				
1				
2				
3				
4				
5				
6				

When we write our numbers, we use a base 10 system. Each digit represents the number of a specific power of 10 the digit contains. For example, the number 28 means that the value of the number is

$$(2 \times 10^{1}) + (8 \times 10^{0})$$
  
 $(2 \times 10) + (8 \times 1)$   
 $20 + 8$ 

We could write our numbers using a different base system but using the same logic. For example, in a base 2 system, 28 is

$$(1 \times 16) + (1 \times 8) + (1 \times 4)$$
  
 $(1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$ 

So in base 2, 28 is 11100.

- 2. How would you write the following base 10 numbers in base 2? Verify on your calculator.
  - a. 57
- b. 71
- c. 8
- d. 39
- e. 17
- 3. How would you write the following base 10 numbers in base 8? Verify on your calculator.
  - a. 57
- b. 71
- c. 578
- d. 39
- e. 100
- 4. How would you write the following base 2 numbers in base 10? Verify on your calculator.
  - a. 110
- b. 1101
- c. 10110
- d. 110111
- e. 1011011
- 5. How would you write the following base 2 numbers in base 8? Verify on your calculator.
  - a. 110
- b. 1101
- c. 10110
- d. 110111
- e. 1011011
- 6. Convert the following base 10 numbers to base 16. Verify on your calculator.
  - a. 54
- b. 121
- c. 300
- d. 5,000
- e. 89,463

NOTE: In base 16, once you use the digits 0 through 9, you then use the letters A through F; therefore, 31 is  $(1 \times 16^1) + (15 \times 16^0)$ . Since there is no single digit for 15, use the letter F (10 = A, 11 = B, 12 = C, 13 = D, 14 = E, 15 = F); therefore, 31 = 1F.

# Lessons to Reinforce/Extend the Grade 8 Science Standards of Learning





# 19. Demonstration on Inclined Planes

Objective Students will determine the mechanical advantage, work input and output, and

efficiency of a simple machine.

**Summary** Students collect data, using three different types of ramps, calculate work input, output,

and percent efficiency for each, and compare the ramps.

Related SOL PS.1, PS.10

**Materials** A board to represent an inclined plane; sandpaper that can be taped onto the board;

wax paper that can be taped onto the board; a wooden block with an eyelet that can be attached to the spring scale and pulled across the board; masking tape; spring scale incremented in Newtons, (one incremented in grams may be used, but then grams must be converted to kilograms, and kilograms to Newtons by multiplying by 9.8); meter stick; copies of attached worksheet; scientific calculators

# Prior knowledge needed

• An understanding of the terms work, work input, work output, mechanical advantage, and percent efficiency

- An understanding of the role friction plays in opposing motion
- The ability to use and read a spring scale accurately
- The ability to recognize and exclude outliers
- The ability to round data

## **Procedure**

- 1. Distribute worksheets, and have the students read the problems.
- 2. Demonstrate the keystrokes to apply the formula M.A. = length/height.
- 3. Demonstrate the keystrokes to apply the formula  $W_o$  or  $W_i = Fd$ , where  $W_o$  stands for work output,  $W_i$  stands for work input, F stands for force in Newtons, and d stands for distance in meters.
- 4. Do the demonstration, as described on the worksheet. You may wish to ask students to volunteer to help with the demonstration.
- 5. Demonstrate keystrokes to find percent efficiency, using 0.24 for the work output and 0.35 for the work input.

Keystrokes:

TI30Xa-SEVA

0.24 ÷ 0.35 2<sup>nd</sup> % =

Casio fx-260 School

0.24 ÷ 0.35 Shift %

Sharp EL-501V and EL-501WBBK

0.24 ÷ 0.35 2<sup>nd</sup> F % =



# Demonstration on Inclined Planes Lab sheet

Name:	Date:
Matching	
Match the terms with the definitions:	
1 Mechanical advantage of an inclined plane	a. Work output
2 The formula for percent efficiency	b. M.A. = length/height
3 The work done by a simple machine	c. Work input
4. The work done on a simple machine	d. $W_0/W_i \times 100$

#### **Materials**

A board to represent an inclined plane; sandpaper that can be taped onto the board; wax paper that can be taped onto the board; a wooden block with an eyelet that can be attached to the spring scale and pulled across the board; masking tape; spring scale incremented in Newtons, (one incremented in grams may be used, but then grams must be converted to kilograms, and kilograms to Newtons by multiplying by 9.8); meter stick; copies of attached lab sheet; scientific calculator

#### **Procedure**

- 1. Attach the wooden block to the spring scale through the eyelet in the block. Hold up the spring scale so that the block hangs freely, and read the scale. Record this number in the output force box in the chart below. If you do not change the block you are using, the output force will stay the same for all three ramps.
- 2. Using the meter stick, measure the length of the ramp and the height of the ramp. Record these numbers in meters in the data chart.
- 3. Place the block at the bottom of the wooden inclined plane. Attach the spring scale to the block. With a consistent force, pull the block up the plane. Read the amount of force it takes to do this; this is the input force. Record this in the data chart as the Trial 1 input force.
- 4. Repeat step 3 two more times, and record the data.
- 5. Average your results. Round to the nearest tenth, and record the average input force.
- 6. Repeat steps 3 through 5, using the sandpaper- and the wax-paper-covered ramps.



# **Data Chart**

Ramp	Ramp	Ramp	Input	Work	Output	Work	%
Description	Length	Height	Force	Input	Force	Output	Efficiency
			T1:				
Plain Wood			T2:				
Plaili WOOU			T3:				
			Avg:				
			T1:				
Candnapar			T2:				
Sandpaper			T3:				
			Avg:				
			T1:				
Way Daner			T2:				
Wax Paper			T3:				
			Avg:				



# **Observations**

# **Calculations**

- 1. Mechanical advantage = length ÷ height
- 2. Work input = input force x ramp length
- 3. Work output = output force x ramp height
- 4. Percent efficiency =  $(W_o \div W_i) \times 100$

Show the work for your calculations by writing the formula, substituting the appropriate terms (including labels) into the formula, solving with your scientific calculator, and then recording your answer with the correct label.

#### Questions

- 1. What does the mechanical advantage of a machine tell you?
- 2. Why is the work input always greater than the work output?
- 3. If you always put more work into a simple machine than you get out of it, what is the advantage of using a simple machine?
- 4. How did the ramp surfaces affect the percent efficiency of the simple machine?
- 5. Can you think of a way to modify the block that would increase the percent efficiency of the ramp? Explain.



# 20. Lab on Mass and Momentum

**Objective** Students will determine the relationships among mass, momentum, and speed.

**Summary** Students roll balls made of different materials down a ramp and collect data on

their comparative speeds. They also roll each ball down ramps into cups and

collect data.

Related SOL PS.1, PS.10

Materials Wooden ramp 1 meter long and with toy car track; 4 textbooks; triple beam

balance or electronic balance; 4 balls of equal volume but different densities (brass, aluminum, wood, and steel); tape measure; stopwatch; large foam cup; flat space for the balls to roll at least 15 meters (e.g., hallway or gymnasium);

copies of attached lab sheet; scientific calculators

### **Procedure**

1. Begin by reviewing the following concepts with the students:

- Using the formula s = d/t to find speed
- Using the formula  $m_0 = mv$  to find momentum
- An understanding of the difference between distance and displacement
- The ability to make line and bar graphs
- The ability to recognize and exclude outliers
- 2. Hand out the lab sheet, and read it over with the class.
- 3. Have students follow the directions on the lab sheet to conduct the experiment and gather their
- 4. After the data has been collected, go through the lab, showing students how to find speed for Part 1 by using *s*=*d*/*t* and fixing the decimal to one place. Doing this will help students see that there is no difference in the speed of each ball. Be sure to account for the minor differences due to timing and/or other variables.
- 5. Have students use the data they collected in Part 2 of the lab to find the average distance that the cup moved, fixing the calculator to one decimal place.
- 6. Have students make the two graphs and answer the questions on the lab sheet.

Keystrokes, using the sample data for the time to roll one meter:

Trial 1: 1.42 sec; Trial 2: 1.39 sec; Trial 3: 1.44 sec

TI30Xa-SEVA

2<sup>nd</sup> | Fix | 1 | fixes the calculator to one decimal place.

1.42  $\Sigma$ + 1.39  $\Sigma$ + 1.44  $\Sigma$ +  $2^{\text{nd}}$   $\overline{x}$  1.4

NOTE: If you use the On/AC button to clear the screen, you will lose your fixed places. You will no longer see the word *fix* on the LCD. You should use the CE/C button to clear the screen and maintain your fixed places.

# Casio fx-260 School

mode 7 1 fixes the calculator to one decimal place.

1.42 Data 1.39 Data 1.44 Data enters all the data.

mode takes you out of fix mode.

# Sharp EL-501V and EL-501WBBK

 $2^{nd}$   $F \leftrightarrow E$  1 fixes the calculator to one decimal place.

1.42 Data 1.39 Data 1.44 Data enters all the data.

 $2^{nd} \mid F \leftrightarrow E \mid$  takes you out of fix mode.



# Lab on Mass and Momentum Lab sheet

Name:	Date:
Part 1: The effect of the mass of an object on its sp	peed rolling down a ramp
Do you believe that heavier objects roll faster down a ramp	? Explain, and make a hypothesis:

#### **Materials**

Wooden ramp 1 meter long and with toy car track; 4 textbooks; triple beam balance or electronic balance; 4 balls of equal volume but different densities (brass, aluminum, wood, and steel); tape measure; stopwatch; large foam cup; flat space for the balls to roll at least 15 meters (e.g., hallway or gymnasium); copies of attached lab sheet; scientific calculators

# **Procedure**

- 1. Make sure your ramp is elevated to a height of 4 textbooks (about 14 cm).
- 2. Place the first ball on the ramp on the toy car track, and allow it to roll 1 meter down the ramp.
- 3. Time how long it takes the ball to roll 1 meter, and record the time in the chart below.
- 4. Repeat steps 2 and 3 twice more with the first ball. Find and record the average time for the three trials with the first ball.
- 5. Repeat steps 2 through 4, using the remaining balls.
- 6. Use the balance to find the mass of each ball. Record this data.

Ball Type	Mass of Ball (g)	Time to Roll (sec)	Average Time (sec)	Average Speed S=d/t (show work)
Wood		T1: T2: T3:		
Aluminum		T1: T2: T3:		
Steel		T1: T2: T3:		
Brass		T1: T2: T3:		

# Conclusion

How did an increase in mass affect the speed of a ball?

# Part 2: The effect of the mass of the ball on the distance it will push a cup.

Do you believe that the more mass an object has, the farther it will push a cup?	_ Explain and
make a hypothesis:	

#### **Procedure**

- 1. Cut a large foam cup in half from top to bottom, and place one half (cut side down) on the floor at the bottom of the ramp so that it is propped on the ramp.
- 2. Allow one of the balls to roll down the ramp and enter the cup.
- 3. Measure the distance in meters that the cup moves from the ramp. (Be sure to measure the distance the cup travels, not its displacement.)
- 4. Make two additional trials per ball, and record the data. Find and record the average distance for each ball.

Ball	Distance Cup Moved (m)	Average Distance (m)
	T1:	
Wood	T2:	
	T3:	
	T1:	
Aluminum	T2:	
	T3:	
	T1:	
Steel	T2:	
	T3:	
	T1:	
Brass	T2:	
	T3:	

## Conclusion

How did the mass of the ball affect the distance the cup traveled?

# Graphing

- 1. Make a line graph of the mass of each ball vs. the average speed of each ball.
- 2. Make a line graph of the mass of each ball vs. the average distance each ball moved the cup.

# Questions

- 1. There are two identical trucks, but one has a full load and the other is empty. Which one will roll faster down the same hill? \_\_\_\_\_\_ Explain.
- 2. How is the experiment you did in Part 2 related to the formula for momentum (mass x velocity = momentum)?

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# 21. Lab on Potential Energy

**Objective** Students will determine the loss of potential energy on an object.

**Summary** Students conduct an investigation on the transfer of potential to kinetic energy

in a bouncing ball. They use a scientific calculator to average data.

**Related SOL** PS.1, PS.5, PS.6, PS.10

**Materials** Scientific calculators; a ball that bounces; meter stick; graph paper

# **Procedure**

1. Begin by reviewing the following concepts with the students:

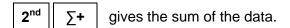
- Introduction to potential and kinetic energy
- An understanding of the Law of Conservation of Energy and how friction affects motion
- Knowledge of statistics terms: *n value*, *summation*, and *x bar* (The advantage of this in science is the ability to add in new data and the ability to see how many pieces of data have been entered.)
- 2. Drop the ball from a specified height, and ask the class why the ball never seems to bounce back to the height from which it was dropped. Students should discuss energy conversions like sound or waste due to friction.
- 3. Hand out lab sheets, and have students read them.
- 4. Hand out materials, and have students gather data. (You might wish to give different groups different types of balls and later discuss the difference this makes.)
- 5. Demonstrate calculator keystrokes for finding the average (see below).
- 6. Have students complete the lab.

# Keystrokes (for example):

Drop Height	Bounce Height Trial 1	Bounce Height Trial 2	Bounce Height Trial 3	Σχ	X	<u>Bh</u> Dh	Decimal	Percent
0.5 m	0.39 m	0.39 m	0.36 m	1.14	0.38	.38/.50	0.76	76%

# TI30Xa-SEVA







$$2^{nd}$$
  $n$  gives the number of terms entered.

# Casio fx-260 School

All Statistics functions are in blue above the keys indicated.

Enter stats mode: | Mode | .

0.39 Data 0.39 Data 0.36 Data enters all the data.

**Shift**  $\sum$ + gives the sum of the data.

**Shift**  $\overline{X}$  gives the mean.

**Shift** *n* gives the number of terms entered.

Exit stats mode: Mode 0

# Sharp EL-501V and EL-501WBBK

Enter stats mode: 2<sup>nd</sup> On/C

0.39 Data 0.39 Data 0.36 Data enters all the data.

**2<sup>nd</sup> F**  $\Sigma$ +  $\overline{X}$  gives the sum of the data.

*n* gives the number of terms entered. It is in blue and does not require **2**<sup>nd</sup> **F** 

Exit stats mode: 2<sup>nd</sup> On/C

key.



# Lab on Potential Energy Lab sheet

Nam	e:					_ '	Jate:				
<b>Topi</b> The		op height o	n bounce h	neight of a b	oall						
<b>Prob</b>		the drop he	eight of a b	all affect th	e perce	nt of e	energy	lost o	n making or	ne bounce?	<b>,</b>
<b>Purp</b> To d		ow much p	otential ene	ergy a ball l	oses or	n maki	ng on	e boun	ce		
	erials ntific calcul	ators; a ba	ll that boun	ces; meter	stick; g	raph p	aper				
Proc	edure										
1.	Drop a ball from the heights shown in meters in column 1 of the table below. After each drop, measure the height the ball reaches after bouncing on the floor, and record the bounce height. For each drop height, do three trials and find the average. Use your scientific calculator to do										
3.	this, and record your $\Sigma x$ value, your $n$ value, and then your average ( $\overline{x}$ ).  Using the average for each height, find the ratio of the bounce height to the drop height ( $\frac{Bh}{Dh}$ ).										
4.	Divide the average bounce height by the drop height to put the ratio into decimal form.										
5.	bounced b		s. Graph th	ese results					original drop heights on		
	Drop Height	Bounce Height Trial 1	Bounce Height Trial 2	Bounce Height Trial 3	Σχ	n	X	Bh Dh	Decimal	Percent	
	0.5 m										]
	1.0 m 1.5 m										-
	2.0 m										•
	2.5 m										]
_	clusion s the height	t of the drop	o affect the	amount of	energy	lost o	n a bo	ounce?	Ехр	lain.	



# Questions

- 1. Why should you use percent rather than the actual bounce height?
- 2. List at least two improvements you could make in this lab.
- 3. If you dropped a ball from a height of 10 meters, predict how high it would bounce, based on the information you got from your experimentation.



# 22. Investigating the Pendulum

**Objective** Students will determine the effect of string length on the period of a pendulum.

**Summary** Students investigate using five different lengths of string for a pendulum and

calculate the percent error.

Related SOL PS.1, PS.6

Materials One meter stick per group of students; 110 cm common household string;

protractor; peg stand with peg; 100 g mass; stopwatch; scissors; graph paper;

scientific calculators; copies of attached lab sheet

#### **Procedure**

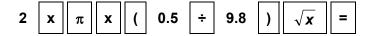
1. Begin by reviewing the following concepts with the students:

- An understanding of potential and kinetic energy
- An understanding of the Law of Conservation of Energy
- 2. Hand out lab sheets, and have the students read them.
- 3. Explain the formula for calculating theoretical time ( $T = 2\pi \sqrt{L/g}$ ), and then demonstrate the keystrokes, if necessary.
- 4. Have the students follow the directions on the lab sheet.
- 5. Demonstrate keystrokes to find percent error, if necessary.

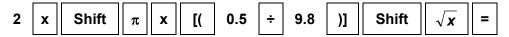
# Keystrokes:

For procedure #2, using a value of 0.5 m for L (length),  $\pi$  for  $\pi$ , and 9.8 m/sec<sup>2</sup> for the value of g (gravity).

TI30Xa-SEVA



Casio fx-260 School



Sharp EL-501V and EL-501WBBK

2 
$$\times$$
 2<sup>nd</sup> F  $\pi$   $\times$  ( 0.5  $\div$  9.8 )  $\sqrt{\phantom{a}}$  =

For procedure #5, using the theoretical time as 1.42, and the actual time as 1.48:

TI30Xa-SEVA

Casio fx-260 School

Sharp EL-501V and EL-501WBBK



# Investigating the Pendulum Lab sheet

Name:	Date:	
<b>Topic</b> The effect of the length of the string on the period of the	e pendulum	
Problem		
Hypothesis		

#### **Materials**

One meter stick per group of students; 110 cm common household string; protractor; peg stand with peg; 100 g mass; stopwatch; scissors; graph paper; scientific calculators; copies of attached lab sheet

#### **Procedure**

- 1. Set up the peg stand so that a weight tied to a peg will move freely back and forth when released from a pre-determined height.
- 2. Attach one end of the 110 cm string to the peg and the other end to the weight (100 g mass) so that the length of the string from peg to weight is 100 cm.
- 3. Lift the weight to a 45-degree angle. Drop the weight while starting the timer. Time how long it takes the weight to make 10 complete periods (back and forth = 1 period). Record this actual time to the nearest thousandth.
- 4. Repeat step 3 two more times.
- 5. Find the average time for 10 periods, and record.
- 6. Repeat steps 2 through 4, using 80, 60, 40, and 20 cm lengths of string from peg to weight. (Cut the 100 cm string shorter each time).
- 7. Complete the data chart.

String Length in cm	Actual Time for 10 Periods (sec)	Average Time for 10 Periods (sec)	Average Time for 1 Period (sec)*	Theoretical Time for 1 Period (sec)	Percent Error (%)
100					
80					
60					
40					
20					

<sup>\*</sup> To find the average time for 1 period, divide your average time for 10 periods by 10.



# **Observations**

#### **Calculations**

- 1. Calculate the theoretical time for one period by using the formula  $T = 2\pi \sqrt{L/g}$ , where T = the theoretical time in sec, L = length of string in m, and g = the force of gravity in m/sec<sup>2</sup>. Round to the nearest hundredth. Don't forget to convert your string length to meters before you do your calculations!
- 2. Calculate your percent error by using the formula: % error = theoretical time ÷ actual time x 100. Round to the nearest whole number. Alternatively, you may fix the decimal point to 0 places and have the calculator round.

# Graphing

Create a line graph of the length of the string vs. the time for 1 period. Graph two best-fit lines, one that represents your actual data, and the other that represents your theoretical data.

Conclusions		

# Questions

- 1. Draw a picture of the lab you did. On the picture, label the following points:
  - Point A the place on the pendulum where the maximum kinetic energy is found
  - Point B the place on the pendulum where the maximum potential energy is found
  - Point C the place on the pendulum where the minimum kinetic energy is found
  - Point D the place on the pendulum where the minimum potential energy is found
- 2. Explain how the Law of Conservation of Energy was demonstrated by this lab. Include in your explanation why the drop point of the mass is always higher than the rebound point. If energy is conserved, why does this happen? Where does the energy "go"?
- 3. Take a look at your percent error. Were you ever 100% accurate? Why, or why not? With what lengths of string were you more accurate? Can you offer any explanation for your findings?